FUJITSU

Delivering Quantum-Inspired Optimization Solutions with Fujitsu's Digital Annealer

White paper

This white paper describes the technology, the methodology and the engagement model of delivering Quantum-Inspired Optimization services using Fujitsu's Digital Annealer (DA). DA is a unique high performance computing system built with a new, world-first quantum-inspired hardware architecture, developed by Fujitsu for solving complex optimization problems.

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Introduction

Solving the most complex, intractable optimization problems by using the quantum power in FUJITSU Quantum-Inspired Computing Digital Annealer paired with Fujitsu's professional services in Quantum-Inspired Optimization and Artificial Intelligence (AI).

Fujitsu's Digital Annealer System (DA) at a Glance

DA is a quantum-inspired technology that enables businesses to solve complex optimization problems. It brings the benefits of quantum annealing while utilizing standard digital circuits with thousands of fully interconnected bits.

DA belongs to the class of specialized high-performance computing systems. It is specialized because it is purpose-built for solving a class of optimization problems, i.e., Combinatorial Optimization problems, which are often unsolvable or can't be solved in a reasonable time and accuracy using traditional computers.

The Digital Annealer technology consists of hardware and software that are tailor-made and integrated to solve complex combinatorial optimization problems. The DA hardware consists of one or more hardware chips built with a new, world-first quantum-inspired hardware architecture developed by Fujitsu. The DA chip fits in the palm of a small hand, and operates in standard room temperatures and infrastructures (unlike the quantum computers that require expensive industrial near absolute zero cooling systems, and still have low fault tolerance). The DA software includes multiple components, including specialized algorithms and Application Programming Interfaces (APIs).

Figure 1 below shows the DA hardware chip.

Figure 1. Digital Annealer Hardware

Solving Combinatorial Optimization Problems

Combinatorial Optimization is the process of finding an optimal solution (minimum or maximum) of a problem that has a discrete (finite) but very large configuration space (set of states). The set of possible solutions is generally defined by multiple restrictions, and the best solution optimizes an objective function, e.g., minimizes cost or maximizes profit. Finding the optimal or even a good solution of these problems using classical computers is extremely difficult and often impossible within a reasonable time or accuracy.

The Combinatorial Optimization problems have a wide variety of applications across multiple industries, both in the public and private sectors. Popular examples include **the knapsack problem, the traveling salesman problem (TSP), Boolean satisfiability (SAT), constrain satisfaction (CSP), maximum cut (MAX-CUT), graph similarity, portfolio optimization, route optimization, scheduling, planning, task allocation, assignment, software validation,** etc.

The traditional approach to solving large Combinatorial Optimization problems is to develop specialized mathematical algorithms and use clever heuristics tailored to the problem at hand. This methodology is not easily extensible to new problems and ever changing practical applications, and requires a lot of time dedicated to research and development.

With the emergence of quantum computing, an effective method for solving Combinatorial Optimization problems has gained interest: **Quantum Annealing**. This method starts from a quantum mechanical superposition of all possible states (candidate states) with equal weights and uses quantum fluctuations to derive the optimal solution as a ground state (output).

Solving problems by Quantum Annealing requires a specialized **quantum computer** and comes with the myriad of complexities including commercial readiness, high price tag, special infrastructure requirements, and error correction overhead. The Canadian company D-Wave Systems, Inc. builds commercially non-universal quantum computers (called quantum annealers) for solving optimization problems through Quantum Annealing.

The DA technology can be used successfully to solve the same problems that are solvable by Quantum Annealing while offering many advantages: small size, standard infrastructure and environment requirements, stability and lower price. Similar to Quantum Annealing, the Digital Annealer system increases the escape probability from the local minimum energy state with parallel state evaluation and it is much faster than traditional simulated annealing techniques. DA also features stochastic parallelism and full interconnectivities of its hardware bits which enables the system to rapidly solve large problems with high precision.

The DA system requires the Combinatorial Optimization problem to be formulated as **Quadratic Unconstrained Binary Optimization (QUBO)**. This is the same input required for Quantum Annealing.

The QUBO model is gaining popularity as a unifying method in solving problems on the Fujitsu's Digital Annealer, the D-Wave Quantum Annealer and the IBM neuromorphic computers (see [3]). A wide variety of Combinatorial Optimization problems can be formulated as QUBO, making the QUBO solvers like DA a preferred choice for developing and implementing a standardized optimization methodology in practice.

The QUBO model advantage is its simplified formulation as a quadratic polynomial over a finite set of binary variables. Many optimization problems can be formulated either directly as QUBO or as a **Higher Order Binary Optimization (HOBO).** The HOBO model expresses an optimization problem as a polynomial with degree higher than two over a finite set of binary variables. The HOBO models can still be solved by QUBO solvers because the HOBO formulations can be re-expressed into QUBO through degree-reduction techniques (see [5]).

In addition, the QUBO model is equivalent to the Ising model which has important applications in physics. The QUBO can be translated to an Ising Hamiltonian through a simple linear transformation. Many NP-hard and NP-complete problems have an Ising formulation. The following classes of problems can be expressed through Ising Hamiltonians and are candidates for QUBO solvers: **partitioning,**

binary integer linear programming (ILP), covering and packing, inequalities, coloring, Hamiltonian cycles, tree, and graph isomorphism problems (see [4]).

In conclusion, while we agree that the quantum computers are the future, we believe that DA will surpass a quantum system for solving Combinatorial Optimization problems for the next decade. In addition, we believe that the current and next generation of DA will still have their place even when Quantum Annealing becomes mainstream due to its small size and versatility.

Quantum-Inspired Optimization with the DA System

The DA system is offered as a Cloud Service or it can be installed on premises.

The DA system offers several QUBO solvers controlled by configuration parameters, e.g., number of iterations, number of replicas, mode, etc. The parameters can be custom selected by the user or used from default.

The input to DA consists of a QUBO model, modeling data, and configuration parameters. The output is the best solution and various model and solution parameters, e.g., the minimized cost / objective function, the best solution variables, the DA running time, the DA number of runs, the DA number of iterations, etc.

The DA system consists of one or more hardware chips. Each DA chip has 8,192 bits that are fully connected, allowing all bits to freely exchange information. The full bit connectivity and the built-in parallel processing allows the DA system to solve large-scale problems. The DA system also features inter-bit coupling which translates into offering up to 64-bit precision (2^{64} gradations) and high accuracy of the solution. In general, Fujitsu's DA system could offer up to 1,000,000 bits and up to 64-bit precision with software partition technology and multichip support.

The number of DA bits maps to the binary variables in the QUBO model. The larger the number of bits the larger the problem (in terms of optimization decision variables) DA can solve in a single run.

The DA precision maps to the coefficients in the QUBO input. The higher the precision the larger coefficients of the optimization problem the DA can solve in a single run.

The number of variables and the precision are connected in the following way: one DA chip can handle a QUBO model with up to 8,192 (2^{13}) variables with 16-bit precision, or a QUBO model with up to 4,096 (2^{12}) variables with 64-bit precision.

Figure 2 below depicts the information flow architecture for solving problems with DA (see [2]).

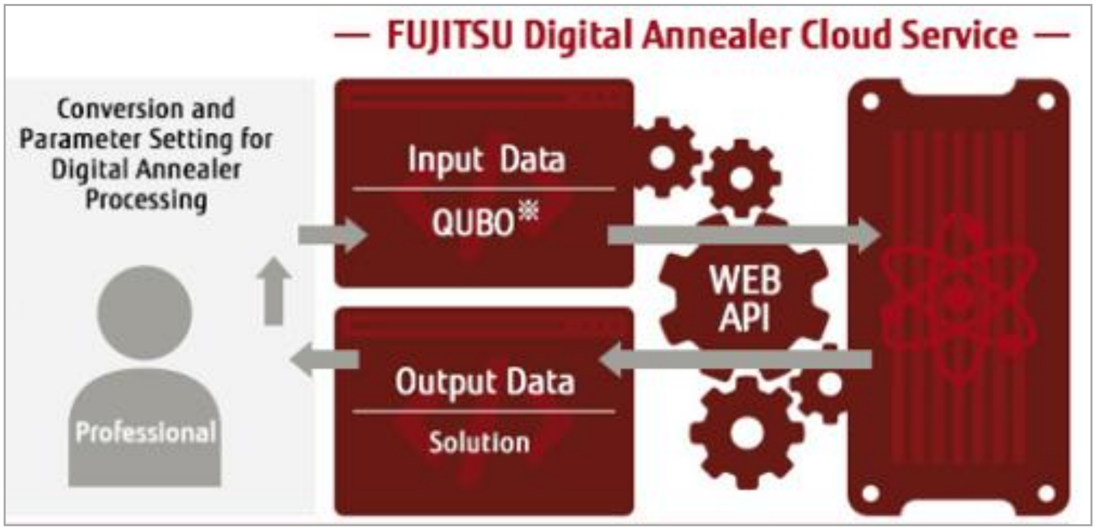

Figure 2. Digital Annealer System – Information Architecture

In the framework of an end-to-end real life solution deployed by corporations, the optimization engine is only a small but critical part of a large scale implementation. Practical advanced analytics include a multi-phased approach and implementation of Internet of Things (IoT), data ingestion, data transformations, data storage, analysis and information delivery. Optimization capabilities are built into the data analysis phase along with advanced analytics, artificial intelligence, reporting and automation capabilities.

An overarching high-level architecture is presented in Figure 3 to illustrate the role of optimization in complex real life implementations.

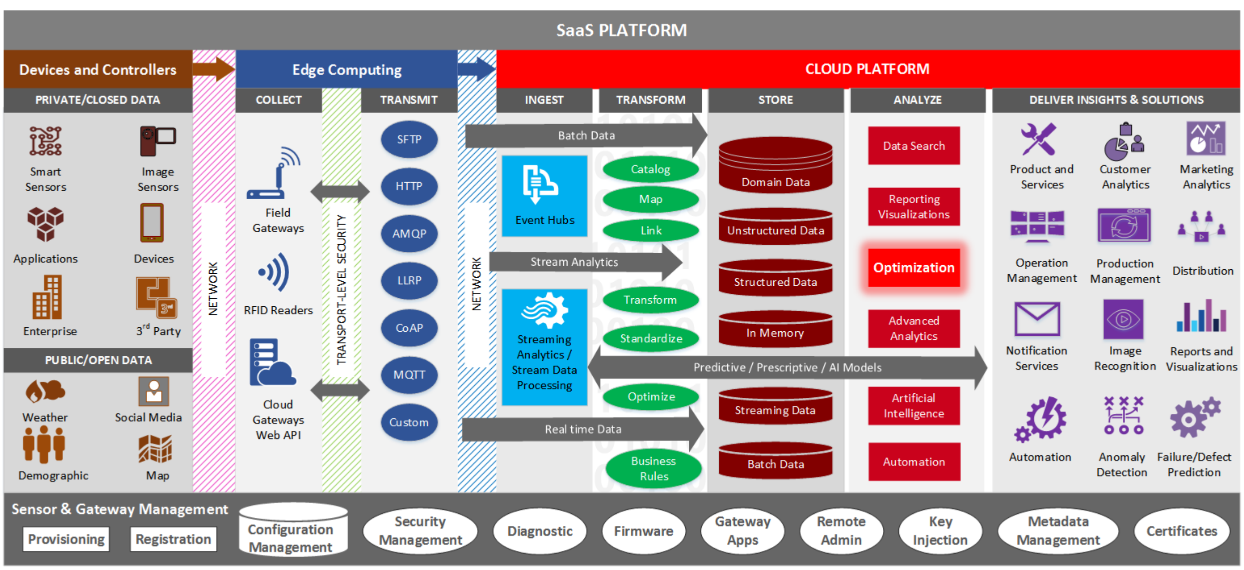

Figure 3. End-to-End High-Level Architecture including Quantum-Inspired Optimization

Quantum-Inspired Optimization Methodology

Fujitsu's team uses a proven Quantum-Inspired Optimization methodology to deliver extensible, flexible and scalable optimization models that expand with the addition of new data sources and capabilities. Our methodology consists of three phases: model, run on DA, and report.

We start with the "Model" phase during which we discuss business problems with our clients, collect and understand the supporting data, develop the conceptual and logical models, and formulate the objective function and the constraints in a mathematical form. We revisit this phase during the entire project to make adjustments to the data and the models as new insights and feedback are gathered from our clients.

When solving large real life optimization problems the solutions need to be scalable and extensible. Using the DA system within an advanced analytics and AI framework requires using of a variety of techniques and algorithms in addition to the mathematical and QUBO formulations of the optimization problems. If the real life problem is very large, we develop an AI framework to scale and automate the solution. We create scaling algorithms to divide the problem into solvable sub-problems and combine the results to derive the best solution and calculate the best value of the objective function.

In the second phase, "Run on DA", we create and code the QUBO formulation in order to map the problem to the DA system. We also select the DA solver and the necessary parameters, and run the model on the DA system to get the best solution. It is in this phase that we execute performance tuning as needed.

The activities in the third phase, "Report", run from the initiation to the conclusion of the project. During this phase we analyze the results, prepare reports with findings and lead frequent discussions with our clients that allow us to fine tune the data and the modeling algorithms. We identify success criteria, metrics and key performance indicators (KPIs) to evaluate the solution and ensure overall success. Last but not least, we prepare interactive BI reports that help with making the solution transparent to a wider audience. The reports illustrate the main features of the solution, and provide visibility to various user controlled parameters. This allows business users to make informed decisions and establish optimal business rules for implementation of the solution in different areas of the business.

Figure 4 below illustrates our Quantum-Inspired Optimization delivery methodology is North America.

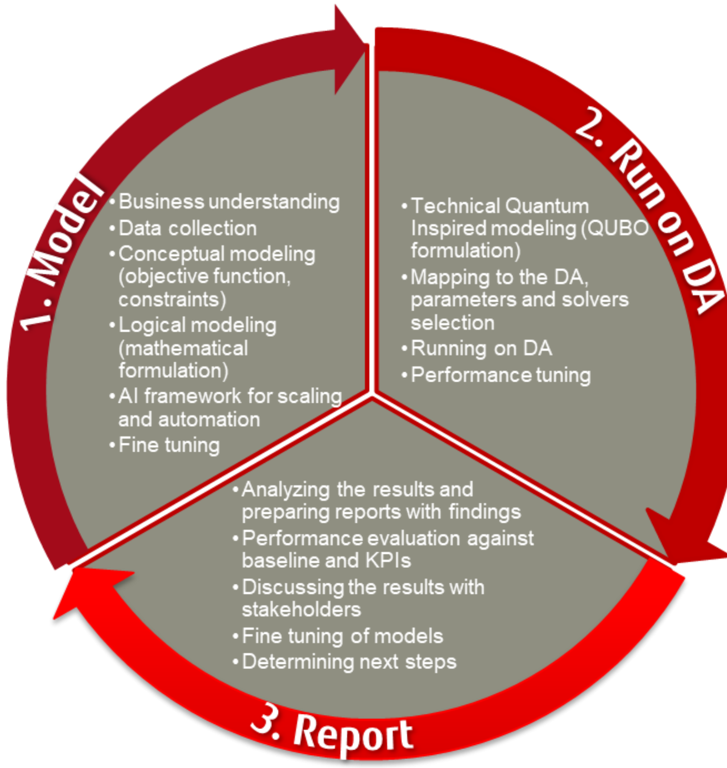

Figure 4. Quantum-Inspired Optimization Methodology for Project Delivery

Quantum-Inspired Optimization Engagement Model

As discussed above, DA is a specialized high performing computing technology for solving Combinatorial Optimization problems that are mathematically formulated as QUBO (Quadratic Unconstrained Binary Optimization) and HOBO (Higher Order Binary Optimization).

The DA technology is offered through cloud services or through custom on-premises installations.

Fujitsu offers a variety of professional services that use the DA system as accelerator to deliver value to clients by solving new emerging optimization problems or improving on existing optimization methodologies.

The differentiators come from the cultural and technological strengths of Fujitsu: continuous innovation, legendary quality, global delivery and responsible business practices.

The engagement model for delivering Quantum-Inspired Optimization solutions starts with a discovery phase that consists of ideation conversations with our clients and continue with a discovery workshop at which we identify business use cases and evaluate the business value and the technical feasibility of the DA technology for solving them. In the next two phases, "Proof of Value" and "Market Test", we develop prototypes and minimal viable products (MVP) to prove the value of the DA system as accelerator for solving real life optimization problems. In the last phase, "Deploy and Expand", we scale, extend and productionize the solutions to fully realize the envisioned business value.

Our engagement model in North America is illustrated in Figure 5.

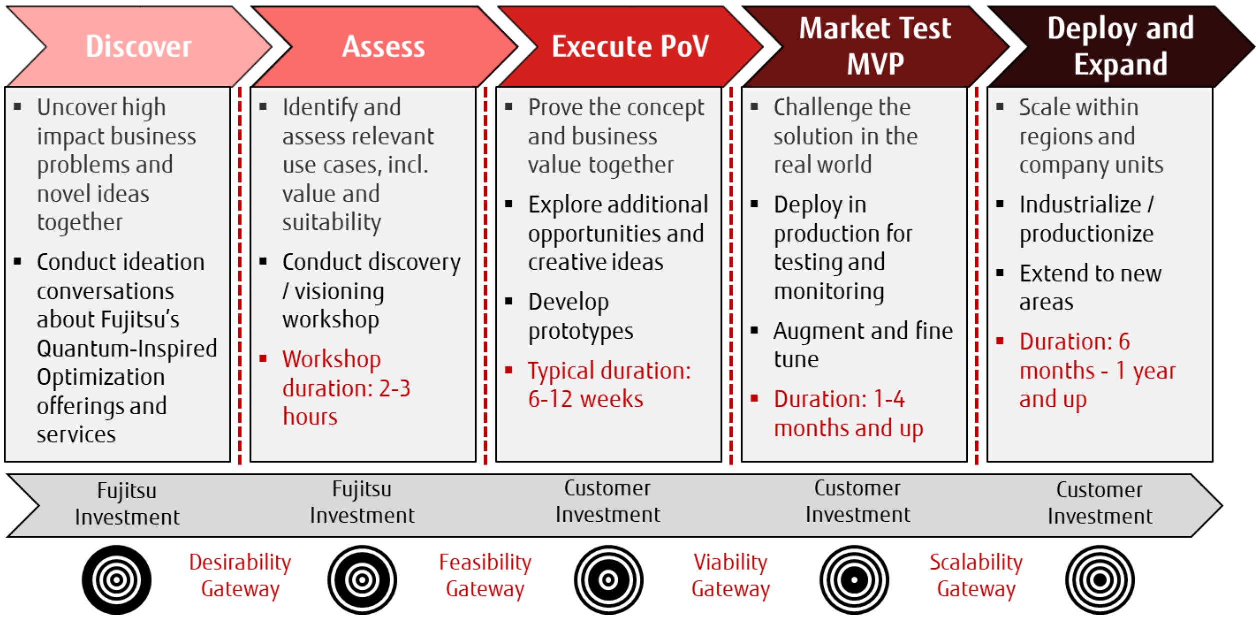

Figure 5. Engagement Model for Quantum-Inspired Optimization Professional Services

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